**Stochastic Demand Models:**

2009, Notes Created by Sil

**What are Stochastic Models?** → Mathematical models involving probability. Probability Distribution is used to represent uncertain factors. Stochastic Models are based on Expected Values (long-run average of all possible outcomes).

If Demand is known → Deterministic Case
If Demand is unknown/uncertain (demand is a random variable) → Stochastic Case

<table>
<thead>
<tr>
<th>3 Types of Stochastic Models:</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) <strong>Single Period</strong> → One time decision (How much to order)</td>
</tr>
<tr>
<td>E.g.: Fashion goods, perishable goods, goods with short lifecycles, seasonal goods.</td>
</tr>
<tr>
<td>2) <strong>Multiple Period</strong> → Periodic decision (How much to order in each period)</td>
</tr>
<tr>
<td>E.g.: Goods with recurring demand but whose demand varies from period to period; Inventory systems with periodic reviews.</td>
</tr>
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<td>3) <strong>Continuous Time</strong> → Continuous decision (Continuously deciding the order quantity)</td>
</tr>
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<td>E.g.: Goods with recurring demand but with variable inter-arrival time between customer orders; Inventory systems with continuous reviews.</td>
</tr>
</tbody>
</table>

(1) **Single Period** Uses “**Newsvendor Model**”

The newsvendor problem has numerous applications for decision making in manufacturing and service industries as well as decision making by individuals. It occurs whenever the amount needed of a given resource is random, a decision must be made regarding the amount of the resource to have available prior to finding out how much is needed, and the economic consequences of having “too much” and “too little” are known.

The Newsvendor Problem:

\[
P(D \leq q) = \frac{C_u}{C_u + C_0} = \frac{Shortage \ Cost}{Shortage + Excess} \]

Service Level to meet the Optimal Amount to Order

Probability of satisfying demand during the period

\[P = \text{Probability} \]
\[D = \text{Random Demand} \]
\[q = \text{Capacity Qty} \]

\[C_u = \text{Cost per unit of being understocked (shortage or unsatisfied demand)}\]
\[= \text{Cost of ordering too little} \]
\[= \text{Selling Price (SP)} - \text{Purchase Price (PP)} = \text{Lost Profit} = \text{Shortage Cost} = \text{Opportunity Cost} \]

\[C_0 = \text{Cost per unit of being overstocked (positive inventory)}\]
\[= \text{Cost of ordering too much} \]
\[= \text{Purchase Price (PP)} - \text{Disposal Price or Salvage Value (SV)} = \text{Loss per excess} = \text{Excess Cost} \]

Optimal Qty to Stock or to Order:

\[q = z\sigma + \mu \]

Find the \(z, \sigma, \mu\) by looking at the Normal Distribution
Normal Distribution:

If the distribution of demand is known to be normal (Normal distribution) with mean $\mu_c$ and standard deviation $\sigma_c$, we use the condition (usually given):

$$ Q^* = P(D \leq q) = \frac{C_u}{C_u + C_o} $$

And then the Optimal Ordering Level is:

$$ q = F^{-1}(Q^*, \mu_c, \sigma_c) \quad \text{OR} \quad q = z\sigma + \mu $$

In Excel: $q = \text{NORMINV}(q, \mu_c, \sigma_c)$

Discreet Distribution

If the distribution of demand is discreet (Discreet distribution), we use the following condition:

$$ q = \min \left\{ q : P(D \leq q) \geq \frac{C_u}{C_u + C_o} \right\} $$

And then the Optimal Ordering Level is:

Look at the table where there are three columns:

<table>
<thead>
<tr>
<th>Demand $(D)$</th>
<th>Frequency</th>
<th>Cumulative frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>15</td>
<td>0.10</td>
<td>0.15</td>
</tr>
<tr>
<td>18</td>
<td>0.15</td>
<td>0.30</td>
</tr>
<tr>
<td>20</td>
<td>0.20</td>
<td>0.50</td>
</tr>
<tr>
<td>22</td>
<td>0.25</td>
<td>0.75</td>
</tr>
<tr>
<td>25</td>
<td>0.15</td>
<td>0.90</td>
</tr>
<tr>
<td>30</td>
<td>0.05</td>
<td>0.55</td>
</tr>
<tr>
<td>35</td>
<td>0.05</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Probability that Demand Qty will be $\leq$ Demand

Find the Cumulative frequency that is the closest to the service level. The corresponding demand will be your Optimal Order Quantity.

The discreet distribution graph looks something like that:

The SUM of all the Frequencies should be 100% always!
Applications for DESC372:

Simple Newsvendor problem

Given:
- SP=$2
- PP=$1
- SV=$0.25

What is the optimal number of newspapers one should buy?

\[ \text{Service Level} = \frac{C_u}{C_u + C_v} = \frac{\text{Shortage Cost}}{\text{Shortage} + \text{Excess}} \]

Shortage Cost = SP – PP \( \rightarrow \) $2 - $1 = $1
Excess Cost = PP – SV \( \rightarrow \) $1 - $0.25 = $0.75

\[ \text{Service Level} = \frac{\$1}{\$1 + \$0.75} = 0.5714 \text{ or } 57.14\% \]

So the optimal order quantity should meet 57.14% of demand.

(Check the cumulative frequency and its corresponding quantity)

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</tr>
<tr>
<td>18</td>
<td>0.10</td>
<td>0.40</td>
</tr>
<tr>
<td>20</td>
<td>0.05</td>
<td>0.45</td>
</tr>
<tr>
<td>22</td>
<td>0.10</td>
<td>0.55</td>
</tr>
<tr>
<td>25</td>
<td>0.10</td>
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<td>0.25</td>
<td>0.90</td>
</tr>
<tr>
<td>40</td>
<td>0.10</td>
<td>1.00</td>
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</table>

The closest to 57.14% is 55%, so the **Optimal Qty to Order** will be 22 newspapers.
Example 15-2: Revenue Management - Allocating capacity to multiple segments

Given:
A trucking company serves two segments of customers.

A. Willing to pay $3.50 per cubic foot but wants to commit to a shipment with only 24 hours notice.
B. Willing to pay $2.00 per cubic foot and is willing to commit to a shipment with up to one week notice.

With two weeks to go, demand for (A) is forecast to be normally distributed, with a mean 3,000 cubic feet and a standard deviation of 1,000.

(a) How much of the available capacity should be reserved for (A)?
(b) How should the company change its decision if (A) is willing to pay $5 per cubic foot?

Rearrange given info

Revenue:
R(A) = $3.50/cubic foot
R(B) = $2.00/cubic foot

Mean(μ) = 3000 cubic feet
Standard deviation(σ) = 1000 feet

Risks:
1) Spoilage: Too much capacity available because we were reserving the space for the higher price buyers
2) Spill: You cannot accommodate the higher price buyers because you already sold your space to lower price buyers.

\[ Q^* = P(D \leq q) = \frac{C_u}{C_u + C_0} \]

\[ C_H = P(C_H \leq D_H) = \frac{P_L}{P_H} = \frac{\text{Price for Lower price segment}}{\text{Price for Higher price segment}} \]

Prob that Capacity reserved for Higher price buyers is ≤ to the actual demand for higher price buyers

Service Level = \[\frac{2.00}{3.50}\] = 0.5714

In Excel
q = NORMINV(Q*, μ, σ)
q = NORMINV(0.5714, 3000, 1000) = 3179.9396
But it has to be the other side so 1-Q*:
q = NORMINV(1 - 0.5714, 3000, 1000) = 2820.0604

Manually
q = zσ + μ
q = 0.18(1000) + 3000 = 3180
But it has to be the other side so the negative of z:
q = -0.18(1000) + 3000 = 2820
So 2820 cubic feet should be reserved for segment (A)

If (A) is willing to pay $5 instead of $3.50:

Service Level = \[\frac{2.00}{5.00}\] = 0.40
q = NORMINV(1 - 0.40, 3000, 1000) = 3253.3471
OR
q = 0.25(1000) + 3000 = 3250
So 3250 cubic feet should be reserved for segment (A) if they are willing to pay $5 instead.
Example 15-5: Revenue Management - Overbooking

Given:
An apparel supplier is taking orders for dresses with a Christmas motif. **Production capacity available** from the supplier is **5,000 dresses**, and it makes $10 for each dress sold. The supplier is currently taking orders from the retailers and must decide on how many orders to commit to at this time. **If it has orders that exceed capacity**, it has to arrange for backup that results in a **loss of $5 per dress**. Retailers have been known to cancel their orders near the winter season as they have better visibility into expected demand.

a) How many orders should the supplier accept if cancellations are normally distributed, with a mean of 800 and a standard deviation of 400?
b) How many orders should the supplier accept if cancellations are normally distributed, with a mean of 15% of the orders accepted and a coefficient of variation of 0.5?

**SOLUTION:**
a) How many should we overbook?

\[
Q^* = P(D \leq q) = \frac{c_u}{c_u + c_o} = \frac{\text{Cost of Wasted Capacity}}{\text{Cost of Wasted Capacity} + \text{Cost of Shortage Capacity}}
\]

\[
S^* = P(C_a \leq O^*) = \frac{\text{Cost of Wasted Capacity}}{\text{Cost of Wasted Capacity} + \text{Cost of Shortage Capacity}}
\]

Prob that Cancellations are going to be \(\leq\) the optimal overbooking level

Service Level = \(\frac{10}{10 + 5}\) = 0.6667

In Excel

\(O^* = \text{NORMINV}(S^*, \mu, \sigma)\)

= NORMINV(0.6667, 800, 400) = 972.3276 dresses

Manually

\(q = z\sigma + \mu\)

\(q = 0.43(400) + 800 = 972\)

So 972 dresses should be overbooked thus

Suppliers should accept the capacity (5,000) + Overbooking limit (972) = 5973 units accepted

b) How many should be accepted with new mean and std deviation?

In Excel

\(O^* = \text{NORMINV}(S^*, \mu, \sigma)\)

= NORMINV(0.6667, 0.15*(5973), 0.075*(5973)) = 1088.9461

Manually

\(q = z\sigma + \mu\)

\(q = 0.43(0.075 * (5973)) + 0.15 * (5973) = 1088\)

So 1088 dresses should be overbooked thus

Suppliers should accept the capacity (5,000) + Overbooking limit (1088) = 6088 units accepted